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Analytical Solution of Missile Terminal Guidance

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A new guidance law that combines pursuit guidance and proportional navigation is proposed. This guidance law depends on two parameters that determine the relative importance of pursuit guidance and proportional navigation. Numerical simulations of the nonlinear equations of motion suggest that the parameters of this law can be chosen to reduce the peak value of the missile acceleration or the duration of the engagement. When the engagement ends in a tail chase, and linearization is valid, the linearized equations of motion lead to a confluent hypergeometric equation. This equation is solved in closed form, in the general case where the target performs maneuvers such that its heading angle is a polynomial function of time. The analytical solution based on linearization and the numerical simulation of the nonlinear equations show good agreement.

Nomenclature

a, b	= parameters in normalized guidance law
C_i	= constants of integration
$f(\cdot)$	= forcing function
h	= altitude
k_i	= coefficients of forcing function
M	= Mach number
$M(\cdot,\cdot,\cdot)$	= confluent hypergeometric function, or Kummer's function
m	= degree of polynomial
n	= velocity ratio
R	= relative distance
r	= normalized relative distance
t	= flight time
$U(\cdot,\cdot,\cdot)$	= confluent hypergeometric function of the second kind
V_M	= missile velocity
V_T	= target velocity
X, Y	= coordinates
z	= modified relative distance
α_i	= coefficients in polynomial of β_p
β	= line-of-sight angle
β_p	= particular solution for β
Γ	= normal acceleration
θ	= missile flight-path angle
θ_T	= target flight-path angle
λ, μ	= parameters of guidance law
τ	= normalized flight time
$(\cdot)_0$	= initial value

I. Introduction

THIS paper considers the standard problem of guiding a craft or a missile on a collision course to a maneuvering target. Among the known guidance laws, the simplest one is pursuit guidance, which constantly directs the velocity of the missile toward the moving target. This law only requires knowledge of the missile–target line-of-sight. Another practical guidance law is proportional navigation, in which the missile detects any rotation of the line-of-sight and applies a turning rate proportional to this rotation. The relative advantages and disadvantages of these guidance laws are well known and presented in many texts.[1–7] Various aspects of proportional navigation are topics of discussion in the current literature.[8–15]

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Guelman[8] obtained a closed-form solution of true proportional navigation for a nonmaneuvering target. Shukla and Mahapatra[9] derived a general form of linearized solution to proportional navigation. Various solutions for pure proportional navigation were given by Mahapatra and Shukla[10] and Becker.[11] Cochran et al.[12] obtained a new closed-form solution in terms of elliptic functions and integrals. Yuan and Chern[13] and Yuan and Hsu[14] also presented closed-form solutions of true proportional navigation for both maneuvering and nonmaneuvering targets and a solution of generalized proportional navigation. Balakrishnan[15] introduced a class of proportional navigation laws through an approximation of the time-to-go and a transformation of the state variables. Gonzalez[4] performed a comparison of pursuit plus proportional navigation with time-varying weighting factors vs conventional guidance laws. This led to recommendations on the numerical values of these factors depending on the range. But there is no previous work that treats a combination of pursuit guidance and proportion navigation with constant navigation constant and constant gain, let alone an analytical solution for that guidance problem.

In this paper we propose a guidance law that combines pursuit and proportional navigation with constant parameters, with the aim of retaining the effectiveness of each guidance law and at the same time reducing the normal acceleration or the duration of engagement. After the formulation of the problem, the effectiveness of the control law is tested numerically for various maneuvers of the target. It is shown that by adjusting two parameters, which control the turning rate of the missile, it is possible to reduce the peak acceleration to an acceptable level or to shorten the intercept time. Furthermore, in the final and most important phase of the interception when the guidance leads to a tail chase, the linearization of the dynamics yields a confluent hypergeometric equation that is solvable analytically when the heading angle of the target is represented by a polynomial function of the independent variable.

II. Equations of Motions

For the case of planar pursuit-evasion, we have the equations

$$\frac{\mathrm{d}R}{\mathrm{d}t} = V_T\cos(\beta-\theta_T) - V_M\cos(\beta-\theta) \tag{1}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{1}{R}[-V_T\sin(\beta-\theta_T) + V_M\sin(\beta-\theta)] \tag{2}$$

We use the flight time t as the independent variable. The geometry of the interception is shown in Fig. 1.

To control the missile toward the intercept point, we use the following new guidance law:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda\frac{\mathrm{d}\beta}{\mathrm{d}t} + \mu\sin(\beta-\theta) \tag{3}$$

The first term on the right-hand side in this guidance equation represents proportional navigation, and the second term represents pursuit guidance. Therefore this guidance law is constructed by combining the two fundamental guidance laws through the use of the two constant parameters λ and μ.

To normalize the equations, the following nondimensional variables and parameters are defined for the case of constant speed of the target. We have

$$\tau = (V_T/R_0)t, \qquad r = R/R_0, \qquad n = V_M/V_T$$
$$a = \lambda, \qquad b = (R_0/V_T)\mu \tag{4}$$

where R_0 is the initial relative distance.

Using these nondimensional variables, the normalized equations are

$$\frac{dr}{d\tau} = \cos(\beta - \theta_T) - n\cos(\beta - \theta) \tag{5}$$

$$\frac{d\beta}{d\tau} = \frac{1}{r}[-\sin(\beta - \theta_T) + n\sin(\beta - \theta)] \tag{6}$$

$$\frac{d\theta}{d\tau} = a\frac{d\beta}{d\tau} + b\sin(\beta - \theta) \tag{7}$$

III. Effectiveness of the Guidance Law

Although we have restricted the analysis to the case of constant speed for the target, the nonlinear equations (5–7) allow its maneuverability through arbitrary changes in the heading $\theta_T(\tau)$ as a function of the dimensionless time and the possibility of modulating the speed ratio $n(\tau)$ for the missile. Of course, the turning rate of the missile is dictated by the guidance law (7), now written in dimensionless form with two arbitrary constants a and b. For this paper, we use a constant value for n.

To show the effectiveness of the guidance law, and the influence of the parameters a and b on its performance, we consider the complete nonlinear system and generate the intercept trajectory numerically.

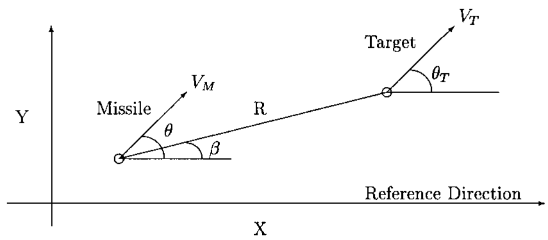

Fig. 1 Geometry of interception.

Without loss of generality, the reference direction is taken along the initial target velocity. Hence, we have at the initial time

$$\theta_T(0) = 0, \qquad \beta(0) = \beta_0, \qquad r(0) = 1, \qquad \theta(0) = \theta_0 \tag{8}$$

In the first two examples, we use a constant speed ratio $n = 3.0$, and $\beta_0 = \frac{1}{20}$, while varying θ_0 from 0 to $\pi/2$. In the last example, we use $n = 1.701$.

In the simplest case, we keep constant the heading of the target. Hence

$$\theta_T = 0 \tag{9}$$

Figure 2 shows the intercept trajectories for various initial headings of the missile, using $\theta_0 = 0, \pi/6, \pi/3$, and $\pi/2$.

As expected, for the same set of values $a = 3.0$ and $b = 3.0$, the tail chase case $\theta_0 = 0$ is the fastest, and as θ_0 increases, it takes longer time for the interception. The interception points are denoted in order of increasing time and range. The normal acceleration Γ, in normalized form, is

$$\Gamma = n\frac{d\theta}{d\tau} = -\frac{an}{r}\sin(\beta - \theta_T) + n\left(\frac{an}{r} + b\right)\sin(\beta - \theta) \tag{10}$$

It is highest at the initial time, and hence its peak value increases when θ_0 increases. To reduce it to an acceptable value for implementation in missiles with low turning rate, as indicated by Fig. 3, we can decrease the value of one parameter to $b = 1.0$, at the expense of a longer time for interception. This is achieved in trajectory 5 in Fig. 2 for the case of $\theta_0 = \pi/2$, with a lower initial turning rate than trajectory 4.

In the next case, we allow the target to have a constant turning rate by using

$$\theta_T = c\tau \tag{11}$$

Figure 4 shows the missile trajectories with the initial values $\theta_0 = 0$ and $\theta_0 = \pi/2$. For the value of $c = 5.0$, both intercept trajectories are achieved with $a = 3.0$, $b = 3.0$, and $n = 3.0$.

In the last example, we consider a sinusoidal change in the heading of the target in the form

$$\theta_T = c\sin\omega\tau \tag{12}$$

The values used for this evasive trajectory are $c = 0.2$ and $\omega = 4.5$. Both intercept trajectories are achieved with $a = 3.0$, $b = 3.0$, and $n = 1.701$. These trajectories are shown in Fig. 5.

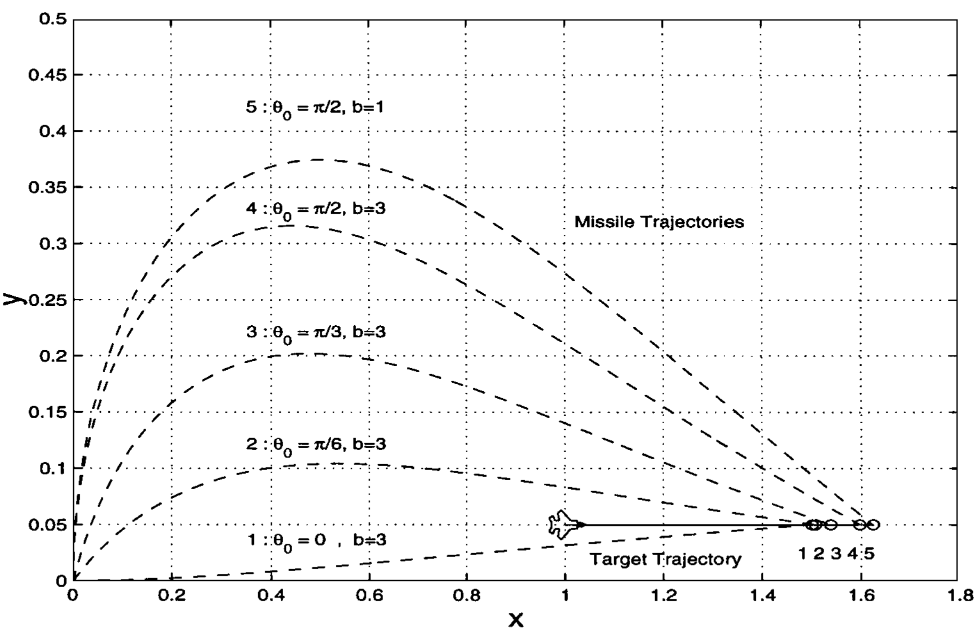

Fig. 2 Intercept trajectories for nonmaneuvering target.

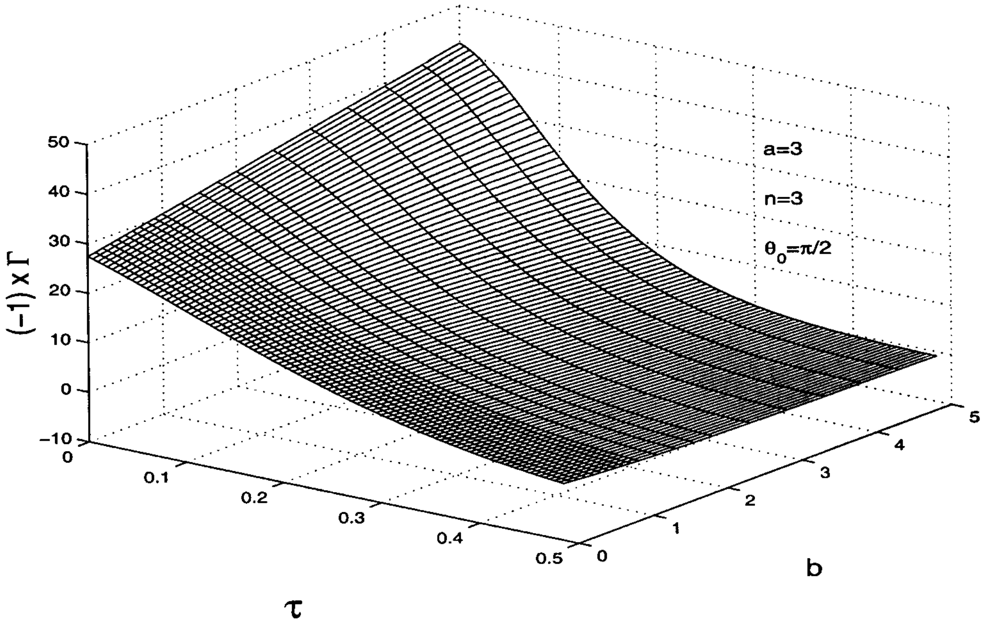

Fig. 3 Time history of normal accelerations vs setting values of parameter *b*.

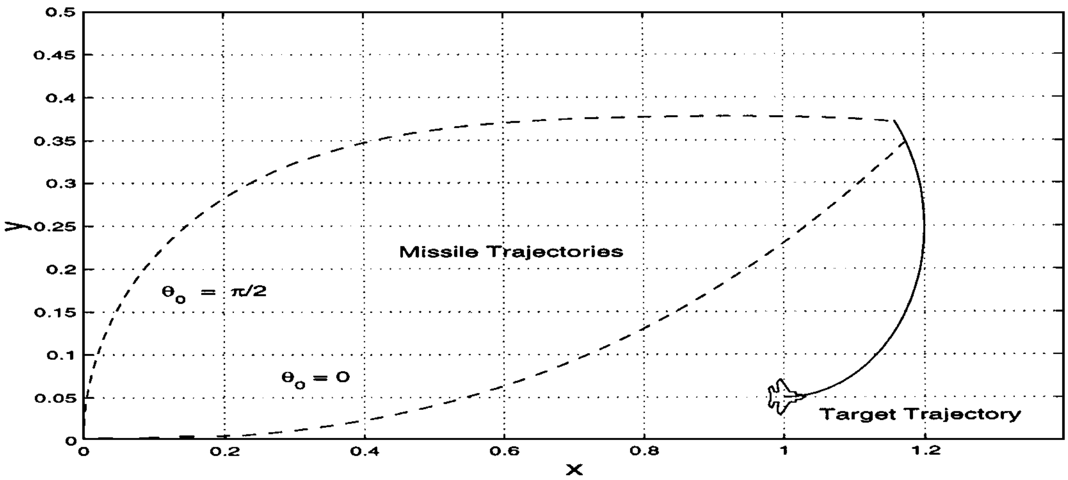

Fig. 4 Circular evasive trajectory.

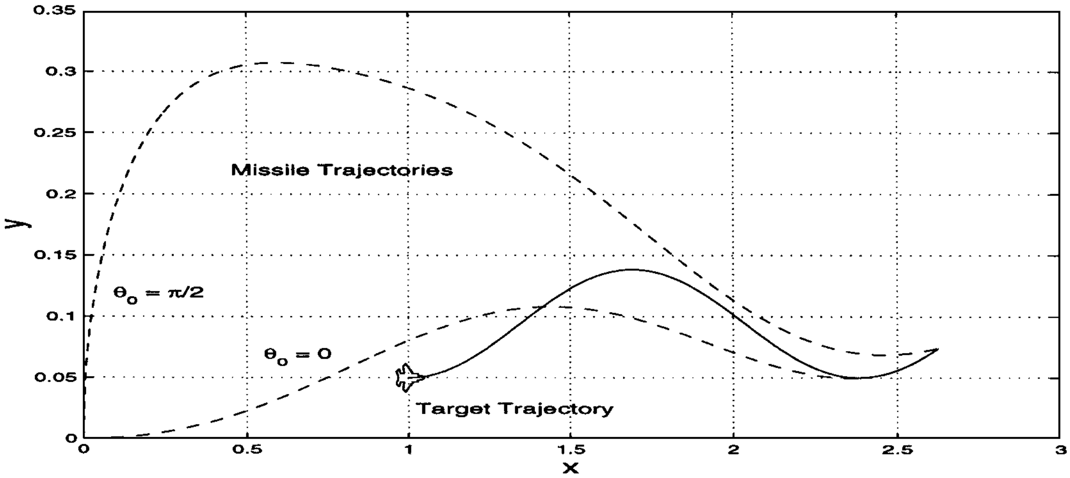

Fig. 5 Sinusoidal evasive trajectory.

We have experimented with a variety of evasive maneuvers for the target, and it has been found that it is possible to choose the parameters a and b to have successful interception with an acceptable acceleration for the missile.

IV. Choice of Parameters a and b

In this section we discuss the choice of the parameters a and b. This guidance law is a combination of proportion navigation and pursuit guidance. Each of these conventional laws, when used alone, provides one degree of freedom in the selection of a parameter. By combining these guidance laws, we have two parameters a and b, which are the navigation constant and pursuit gain. This allows a design tradeoff between the advantages and disadvantages of these laws. As can be seen in Fig. 2, if we select larger values of b, we tend to have a shorter time of interception. This would, however, induce larger accelerations in the guidance. Conversely, if we select smaller values of b, we can reduce the peak value of the missile acceleration, as can be seen in Fig. 3, at the expense of a longer duration of engagement. From another point of view, pursuit guidance tends to be more robust against noise than proportional navigation, but usually proportional navigation follows target maneuvers better than pursuit guidance.[3,4,7] We can trade off these characteristics by a proper selection of a and b, depending on the engagement.

As an illustration, we have shown in Fig. 6 a case that is opposite to the case shown in Fig. 2. Here, we use $\beta_0 = 45$ deg, $\theta_0 = 40$ deg, $\theta_{T0} = 0$ deg, $a = 3$, and $n = 3$. In this engagement, the intercept times with parameter $b = 0$, 1.5, and 3 are 0.4539, 0.4542, and 0.4546, respectively. Therefore larger values of b delay the intercept, whereas from the bottom panel, the initial normal acceleration is reduced by the choice of a larger value of b.

A typical engagement is shown in Fig. 7. Here we have a fighter's evasive maneuver to avoid the threat of a missile interception from behind. In this situation, usually the pilot makes a break turn toward the direction of the incoming missile at the last moment.[16] We set up the physical data as $R_0 = 1464$ m ($h = 10{,}000$ m), $V_T = 0.8M$, and $V_M = 2.4M$ ($n = 3$). The initial condition is $\beta_0 = \theta_0 = 30$ deg and $\theta_{T0} = 0$. The target is making an 8-g right turn against the incoming missile from the 5 o'clock direction. After normalization, Fig. 7 (top panel) shows the trajectories of the target and the missile with $a = 3$, $n = 3$, and $b = -2, 0$, and 2. Their intercept times are 0.4332, 0.4338,

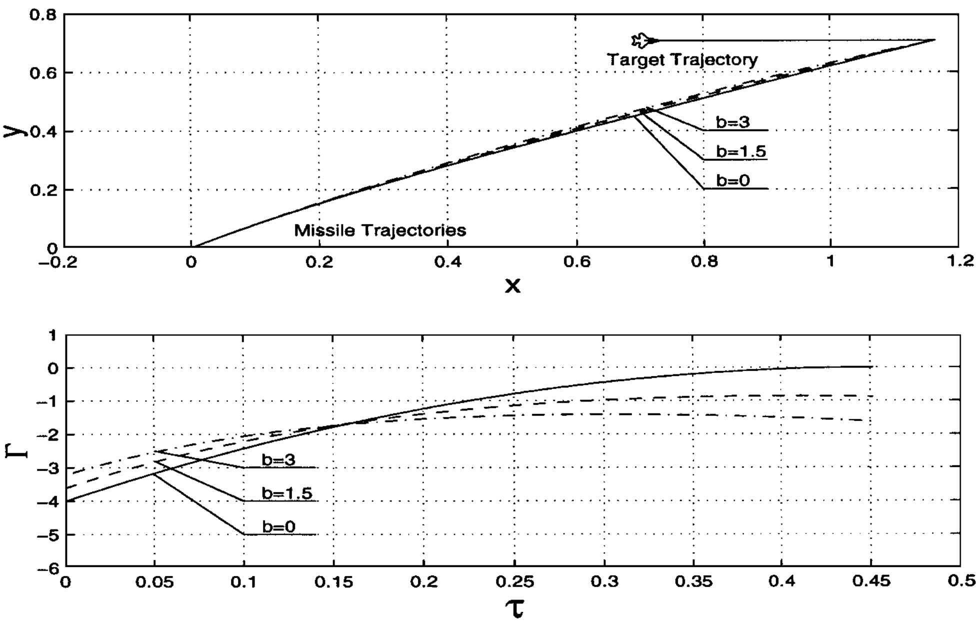

Fig. 6 Intercept trajectories (top) for nonmaneuvering target with various values of b and their normal accelerations (bottom).

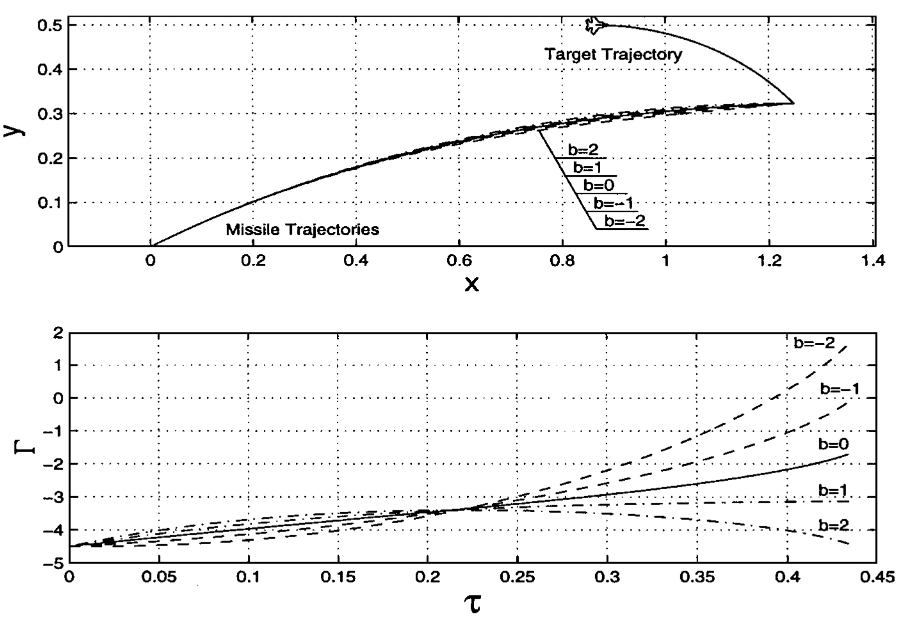

Fig. 7 Intercept trajectories (top) for maneuvering target with various values of b and their normal accelerations (bottom).

and 0.4345, respectively. The normal acceleration of the missile is shown in Fig. 7 (bottom panel). According to these results, if we select negative values of the parameter b, we have a shorter duration of engagement, and the parameter $b \simeq -1.0$ minimizes the final normal acceleration. In this case, $b \simeq -1.0$ should be the preferred value to be chosen if we have already set the parameter $a = 3$.

The pursuit guidance component, with parameter b, of this guidance law tends to adjust the missile flight path onto the line-of-sight direction, whereas the proportional navigation component, with parameter a, attempts to follow the target maneuvers. Notice that negative values of b can be chosen to achieve smaller values of the final acceleration, as shown in Fig. 7 (bottom panel). In general, the optimal choice of parameters a and b, which is important for the performance of the guidance law, is highly dependent on the particular engagement.

V. Terminal Guidance

The control law used in this guidance tends to align the velocity of the missile with the line-of-sight and with the velocity of the target. Hence, the guidance law tends to tail chase in the terminal phase if the missile is approaching from behind the target, and then the linearization of Eqs. (5–7) is valid. In the case where, in the final phase of the homing, the two angles $(\beta - \theta)$ and $(\beta - \theta_T)$ are reasonably small, we have the linearized system for terminal guidance:

$$\frac{dr}{d\tau} = (1 - n) \tag{13}$$

$$\frac{d\beta}{d\tau} = \frac{1}{r}[-(\beta - \theta_T) + n(\beta - \theta)] \tag{14}$$

$$\frac{d\theta}{d\tau} = a\frac{d\beta}{d\tau} + b(\beta - \theta) \tag{15}$$

The equation for r is decoupled, and upon integration we have the linear relation

$$r = 1 - (n - 1)\tau \tag{16}$$

For the integration of the other two equations, we change the independent variable from τ to r to have the new system

$$\frac{d\beta}{dr} = -\frac{1}{r}\beta + \frac{n}{(n-1)r}\theta - \frac{1}{(n-1)r}\theta_T \tag{17}$$

$$\frac{d\theta}{dr} = -\frac{a(n-1) + br}{(n-1)r}\beta + \frac{an + br}{(n-1)r}\theta - \frac{a}{(n-1)r}\theta_T \tag{18}$$

For the integration of this system, it is convenient to use a new independent variable

$$z = br/(n - 1) \tag{19}$$

We notice that because r varies from 1 to 0, this new variable monotonically decreases from $b/(n - 1)$ to 0. By changing to z, Eqs. (17) and (18) can be rewritten as

$$\frac{d\beta}{dz} = -\frac{1}{z}\beta + \frac{n}{(n-1)z}\theta - \frac{1}{(n-1)z}\theta_T \tag{20}$$

$$\frac{d\theta}{dz} = -\frac{a + z}{z}\beta + \frac{an + (n-1)z}{(n-1)z}\theta - \frac{a}{(n-1)z}\theta_T \tag{21}$$

By eliminating θ between these two equations, we obtain a linear second-order differential equation with a forcing function for the variable β:

$$z\frac{d^2\beta}{dz^2} + \left(2 - \frac{an}{n-1} - z\right)\frac{d\beta}{dz} + \frac{1}{(n-1)}\beta = \frac{1}{n-1}\left(\theta_T - \frac{d\theta_T}{dz}\right) \tag{22}$$

Upon obtaining the solution for β, the guidance angle θ for the missile is obtained from

$$\theta = \frac{(n-1)}{n}\left(\beta + z\frac{d\beta}{dz} + \frac{1}{n-1}\theta_T\right) \tag{23}$$

The linear equation (22) is a confluent hypergeometric equation.[17] Including the forcing function on the right-hand side, its general solution is

$$\beta(z) = C_1 M\left[-\frac{1}{n-1}, \frac{2(n-1) - an}{n-1}, z\right] + C_2 U\left[-\frac{1}{n-1}, \frac{2(n-1) - an}{n-1}, z\right] + \beta_p \tag{24}$$

where $M(\cdot,\cdot,\cdot)$ and $U(\cdot,\cdot,\cdot)$ are linearly independent Kummer's solutions of the homogeneous equation and $\beta_p(z)$ is a particular solution. Using this solution into Eq. (23), we have the solution for θ:

$$\begin{aligned}\theta(z) = {}& C_1\frac{n-1}{n}\left\{M\left[-\frac{1}{n-1}, \frac{2(n-1) - an}{n-1}, z\right] - \frac{z}{2(n-1) - an}M\left[1 - \frac{1}{n-1}, 1 + \frac{2(n-1) - an}{n-1}, z\right]\right\} \\ & + C_2\frac{n-1}{n}\left\{U\left[-\frac{1}{n-1}, \frac{2(n-1) - an}{n-1}, z\right] + \frac{z}{n-1}U\left[1 - \frac{1}{n-1}, 1 + \frac{2(n-1) - an}{n-1}, z\right]\right\} \\ & + \frac{n-1}{n}\left(\beta_p + z\frac{d}{dz}\beta_p + \frac{1}{n-1}\theta_T\right)\end{aligned} \tag{25}$$

The constants of integration C_1 and C_2 are determined by the initial conditions on β and θ.

For a maneuvering target, when the heading θ_T is an arbitrary function of τ, we model it as a polynomial in τ, and ultimately through the change of independent variable by Eqs. (16) and (19), we have θ_T as a polynomial in the independent variable z. In this case, the forcing function $f(z)$, which is the right-hand side of Eq. (22), is also a polynomial in z, and the particular function $\beta_p(z)$ in the solutions (24) and (25) for β and θ can be obtained by identification.

As an example, we consider the case of a third-degree polynomial

$$f(z) = k_0 + k_1 z + k_2 z^2 + k_3 z^3 = \frac{1}{n-1}\left(\theta_T - \frac{d\theta_T}{dz}\right) \tag{26}$$

The particular solution of the nonhomogeneous equation (22) is assumed to be a polynomial of the same degree:

$$\beta_p(z) = \alpha_0 + \alpha_1 z + \alpha_2 z^2 + \alpha_3 z^3 \tag{27}$$

where the coefficients α_i $(i = 0, 1, 2, 3)$ are to be found. Upon substituting into Eq. (22) and equating the coefficients of the same power in z, we have the solution

$$\alpha_3 = -\frac{k_3}{A+3}, \qquad \alpha_2 = \frac{3(B+2)\alpha_3 - k_2}{A+2}, \qquad \alpha_1 = \frac{2(B+1)\alpha_2 - k_1}{A+1}, \qquad \alpha_0 = \frac{B\alpha_1 - k_0}{A} \tag{28}$$

where

$$A = -\frac{1}{n-1}, \qquad B = \frac{2(n-1) - an}{n-1} \tag{29}$$

It is not difficult to show that, if the forcing function is a polynomial of degree m, then in the solution for $\beta_p(z)$ for the general case

$$\beta_p(z) = \alpha_0 + \alpha_1 z + \alpha_2 z^2 + \alpha_3 z^3 + \cdots + \alpha_m z^m \tag{30}$$

we first calculate the coefficient α_m

$$\alpha_m = -[k_m/(A + m)] \tag{31}$$

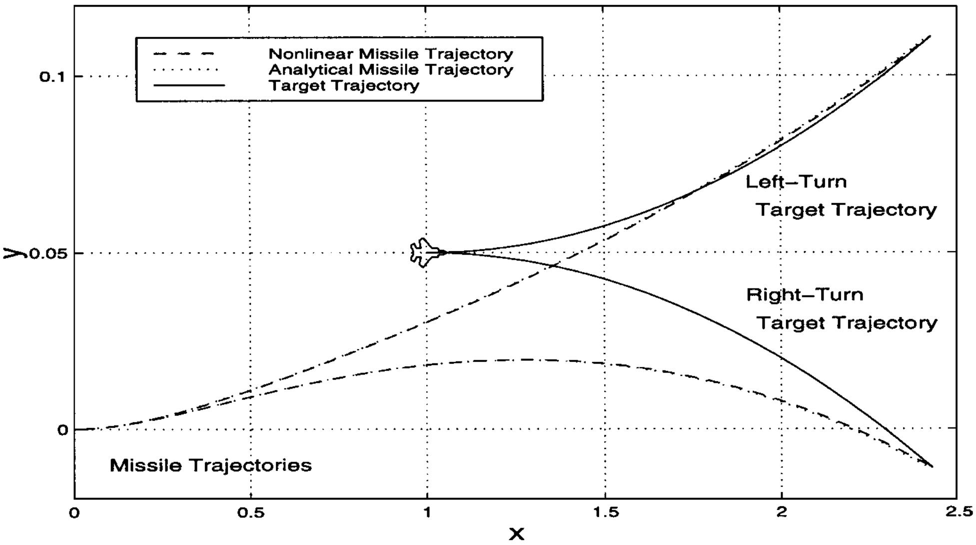

Fig. 8 Circular evasive trajectory.

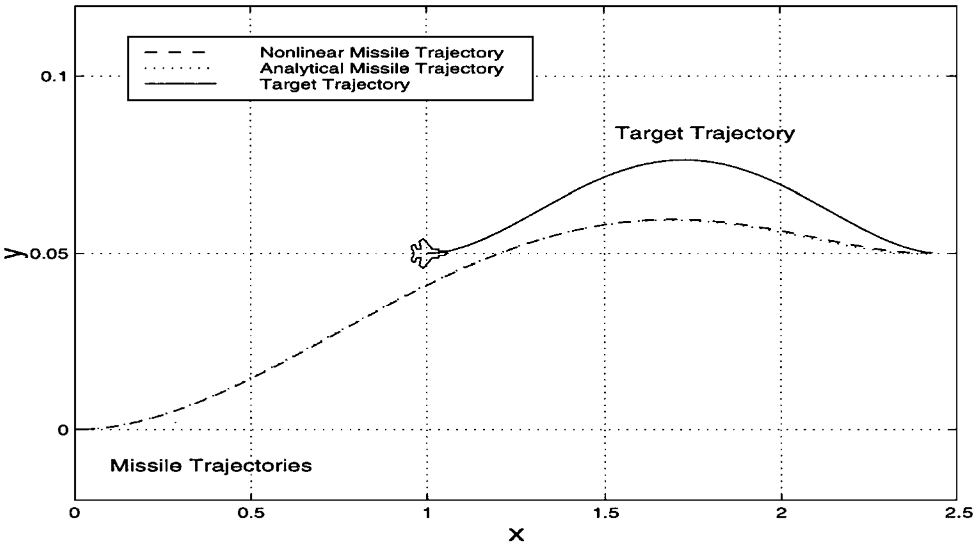

Fig. 9 Sinusoidal evasive trajectory.

and for any other coefficient α_q ($q = 0, \ldots, m - 1$), we use the recursive formula

$$\alpha_q = \frac{(q+1)(B+q)\alpha_{q+1} - k_q}{A+q} \tag{32}$$

As an example, we consider the case of a constant turning rate for the target as given in Eq. (11). Then, in terms of z, we have

$$\theta_T = c\{[1/(n-1)] - (z/b)\} \tag{33}$$

The solution $\beta_p(z)$ is also a linear function, and the coefficients α_1 and α_0 are easily calculated from Eqs. (31) and (32). With $n = 1.701$, initial values $\theta_0 = 0$ and $\beta_0 = \frac{1}{20}$, and using again $a = 3$ and $b = 3$, we generate two evasive trajectories for $c = 0.06$ and -0.06. We calculate the intercept trajectories, first by integrating the nonlinear equations and then by using the analytical solution of the linearized system. The two solutions are in excellent agreement with each other, as seen in Fig. 8.

In the next example, we model the heading change of a sinusoidal evasive trajectory as a polynomial in τ:

$$\theta_T(\tau) = 0.38\tau^3 - 0.827\tau^2 + 0.4\tau \tag{34}$$

After changing the independent variable, again, using the value $n = 1.701$, $a = 3$, and $b = 3$ with the trinomial in z:

$$\theta_T(z) = -0.01407z^3 - 0.08880z^2 - 0.12010z - 0.00928 \tag{35}$$

for the heading of the target, the analytical solution obtained is in perfect agreement with the numerical solution of the nonlinear system. This is shown in Fig. 9.

Finally, note that the extension of this guidance law to the three-dimensional case is possible. But the resulting solution of the linearized system cannot be directly applied to the three-dimensional case because we assumed a planar engagement for that linear solution. This solution agrees well with the nonlinear one when $(\beta - \theta)$ and $(\beta - \theta_T)$ are within ± 10 deg. Further analysis by using the analytical solution, such as miss analysis and optimal choice of guidance parameters and velocity ratio, will be the topic of future work on this guidance problem.

VI. Conclusions

In this paper, a new guidance law that combines pursuit guidance and proportional navigation has been proposed. This guidance law depends on two constant parameters that determine the relative importance of pursuit guidance and proportional navigation. Numerical simulations of the nonlinear equations of motion have been presented and suggest that the parameters of this law can be chosen to reduce the peak value of the missile acceleration or the duration of the engagement. An analysis of the terminal portion of the trajectory has been presented, leading to a confluent hypergeometric equation. A closed-form solution of this equation has been obtained in the general case where the target performs maneuvers such that its heading angle is a polynomial function of time. The analytical solution based on linearization and the numerical simulation of the nonlinear equations show good agreement.

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